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SINGLE-NUCLEON DAMPING

OF

COLLECTIVE VIBRATIONS*†

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# SINGLE-NUCLEON DAMPING OF COLLECTIVE VIBRATIONS*

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This paper reports on a Ph.D. thesis by M. Bauer at the University of Maryland. Further details can be found in the thesis[1], which has been carried out along lines already sketched by the present author[2]. Similar considerations have also been expressed by N. Austern[3], C. Shakin[3], G. Opat[3], and others. In order best to motivate the present discussion it is useful to look at some recent data which has been obtained at the University of Virginia by Bolen and Whitehead[4] last summer. They have measured the ($\gamma$-n) cross section in $O^{16}$ over the giant dipole range of energy and the results are shown in Figure 1, which is taken directly from their paper. One notes that the cross section has pronounced maxima in the region between 20 and 25 MeV, where most of the cross section is located. This is, of course, the giant dipole resonance, but it is interesting to note that in oxygen the giant dipole resonance is split into two peaks. (This results from the spin-orbit force and not from surface deformation.) These two maxima fall at about 22 and 24 MeV. Customary calculations of the shell model have as a goal the computation of these numbers, which are associated with presumed stationary states of the nucleus. Clearly from Figure 1 the situation is more complicated than this and what we are dealing with is a function of the $\gamma$-ray energy and not simply a discrete set of numbers. Therefore we must specify more about the function than simply the locations of its maxima. What we set for ourselves then is the next step in the

specification of the function, namely the calculation of the breadths of the various maxima.

The extended shell model[5] can be applied directly to this calculation, which actually is not much more complicated than the more familiar calculations which limit themselves to the eigenvalues themselves. We are dealing with a situation where the basis states in the shell model are of two different types: (a) bound states, shown in Figure 2 as lying below the threshold at zero energy for particle emission and (b) unbound, shown in Figure 2 as lying in continua associated with the various angular momenta s, p, d, etc. (Here for the moment we neglect the spin-orbit coupling.) Now the first s-state in $O^{16}$ is so tightly bound that it does not enter into discussion, so that we are dealing with excitations out of the filled p level into s- or d-states. The usual shell model calculations concern themselves only with excitations into the first unoccupied s- or into the first d-state. These are the lowest lying states, and are bound to the nucleus.

In order to take into account the damping, all we have to do is to include excitations into all of the different possible s- and d-states. Thus we should consider Feynman diagrams for the time history of the nucleus, such as shown in Figure 3. Here the excitation of a particle into a bound state is shown by a light forward-going line in the direction of increasing time, whereas an excitation into a continuum state is shown by a heavy forward-going line. The chain of bubbles expresses, of course, the well-known picture that the oscillation consists of a resonance among the various relatively degenerate single-particle excitations.

Any one particle may happen to possess the excitation of the nucleus and then pass the excitation on to another particle at the same time jumping back into its natural state and annihilating the associated hole of the particle-hole bubble. Thus we see that the difference between the present approach and the more conventional calculations lies in the inclusion of bubbles containing a heavy line. This changes the problem sufficiently that one might be inclined to abandon all of the previous work on the shell model treatment of the giant dipole resonance in $O^{16}$ and start afresh. However, we prefer a more conservative perturbation-type treatment which starts with the conventional wave function omitting the effect of the continuum single-particle states, and then including them in the next step as a weak perturbation.

This procedure is illustrated in Figure 4, where we schematically indicate the Hamiltonian matrix as acting in two different portions of Hilbert space. The portion of the matrix labeled S.M. includes all matrix elements within the subspace of Hilbert space spanned by the bound state wave functions. H' consists of off-diagonal matrix elements coupling the bound states with the continuum states. The remaining portion of the matrix consists of diagonal and off-diagonal elements relative to the continuum states. Here we make the simplifying approximation of setting all of the off-diagonal elements between continuum states equal to zero. This can be justified on physical grounds. Once a nucleon is excited into a free state, we can expect it to leave the nucleus and not to react again. This situation is expressed by vanishing off-diagonal matrix elements.

To begin the calculation we at first imagine H' to equal zero, and diagonalize the S.M. portion of the Hamiltonian matrix. This is, of course, just the work of Elliott and Flowers[6] and gives us five eigenstates of angular momentum unity, with five corresponding eigenvalues. Two of these fall at about 22 and 25 MeV and are associated with the prominent maxima shown in Figure 1. Having now explicit eigenstates, and for the sake of definiteness let us restrict ourselves to the 22-MeV eigenstate, we must compute the rate of transition resulting from the non-vanishing values of H' between the 22-MeV eigenstate of S.M. and the continuum single-particle excitations.

The rate of decay of transition is given by the standard golden rule of time-dependent perturbation theory:

$$\Gamma = 2\pi \rho \, |\langle H' \rangle|^2 , \tag{1}$$

where $\rho$ is the density of final states and $\langle H' \rangle$ is the bound-free matrix element of H'. This formula has been applied to the computation of all of the various partial breadths of the damping, using a square well nuclear potential for the single-nucleon wave functions. The results of the computations of Bauer[1] are given in Table 1 for the 22-MeV state. It will be noted that the total neutron breadth is of the order of 0.6 MeV and the total proton breadth about twice that. Thus the total breadth of the line comes out to be 1.8 MeV, in encouragingly good agreement with the breadth estimated by Bolen and Whitehead of 1.7 MeV. It is worthwhile to note that this is just twice the imaginary part of the complex energy eigenvalue of the excited state of the nucleus. Thus the 22-MeV state, because of the continuum wave functions, is

shifted by an imaginary amount of 0.9 MeV. Now we should remember a general property of perturbation theory and of Feynman diagrams: the real and imaginary parts of the second-order energy shift are generally of the same order of magnitude. Thus we may expect that second-order perturbation theory, passing through the virtually excited continuum states and back down to the bound states, will give a real shift in the energy eigenvalue also of the order of 1 MeV. Thus one should take the results of the work of Elliott and Flowers[6] and of Brown and collaborators[7], where a good fit is found to the photonuclear maxima, with a grain of salt. Clearly much of this success has been achieved simply by varying parameters until a good fit was obtained, and the inclusion of higher order effects, such as discussed here, can be expected to spoil the fit by the order of 1 MeV.

At this point it is necessary to take note of the fact that the cross section curve such as shown in Figure 1 is a more complicated one than can be described by the term "Lorentzian line." The low energy side does not have the usual Lorentzian tail and therefore Bolen and Whitehead were forced to estimate the width from the full-width at half maximum, measured on a Gaussian fit since a Lorentzian fit was not possible. The lack of the Lorentzian tail can be attributed to an interference between the outgoing single-nucleon wave coming from the damping of the collective state (excited by having the $\gamma$-ray first cause a p-shell nucleon to jump into one of the bound d- or s-waves) and the single-nucleon waves resulting from direct transitions from the p-shell into one of the continuum s- or d-waves. Let us designate the matrix element of the dipole operator for the excitation of the

collective state by D and for the direct excitation into the continuum by D'. If the real part of the energy eigenvalue is E and the imaginary part $-\Gamma/2$, then we have the following resonance formula

$$\sigma(\gamma,n) \alpha \left| D' + \frac{\langle H' \rangle D}{\hbar\omega - E + i\Gamma/2} \right|^2 . \tag{2}$$

In this equation, if there is no direct transition into the continuum ($D'=0$), then we have a pure Lorentzian shape for the cross section, or the standard Breit-Wigner formula. On the other hand we have no reason to expect D' to vanish, and we have a situation already studied in the atomic case by Fano and Prats[8]. As they showed, considerable distortion can result from the interference between direct and intermediate transitions. We can see this from Eq. (2) if we imagine a $\gamma$-ray energy $\hbar\omega$ considerably below the collective resonance. Then the second term inside the absolute value signs is negative and we have a destructive interference, giving rise to the low energy cut-off of the resonance line. This is illustrated in Figure 5, taken from their paper, for energy-independent matrix elements. On the other hand, for $\gamma$-ray energies above resonance, we have constructive interference leading to a very strong tail extending up to higher frequencies. This seems to be evident in connection with the 25-MeV resonance which we see does not come down to 0 above 25 MeV but instead extends on up to 30 MeV. At this energy the cross section still retains a strength of about one-half to one-third of its maximum value.

Now we come to the most controversial part of the work, in which we must distinguish clearly between the resonance effect for $\gamma$-rays, or in other words the collective resonance arising from the interactions among all the particles in the nucleus, and on the

other hand the standard quantum mechanical resonance of a single nucleon in a potential well. If we look at Eq. (2), we see that we should expect maxima for both of these cases, namely whenever $\hbar\omega = E$ and we strike a collective resonance of the system, or also when D' is especially large. We can expect D' to be large when the continuum particle has the right energy to be in single-particle resonance for the attractive well in which it moves. For this case its wave function inside the nucleus will be maximum and all matrix elements computed with this wave function, such as D', will also exhibit a maximum. Thus we should expect an additional bump in the ($\gamma$-n) cross section, and some recent experiments seem to exhibit such a feature. This would be especially true for the ($\gamma$-p) reaction, since all three partial waves ($s_{1/2}$, $d_{5/2}$, and $d_{3/2}$) should have resonances in the continuum, and thus give three additional bumps in the cross section curve for ($\gamma$-p).

It is worthwhile at this point to describe an analogy which is sometimes employed as evidence against the correctness of the idea in the preceding paragraph. One imagines that the zero-order single-particle excitations are analogous to the natural vibrational excitations of a set of uncoupled oscillators. The interaction of the nucleons in the nucleus which produces a mixing of the zero-order states and a shift of the collective resonance line is imagined to correspond to the introduction of coupling among the oscillators. Consequently the normal mode frequencies of the oscillators are shifted. This is illustrated in Fig. 6 which shows a typical behavior of some linear response function for a system of two oscillators. The solid curves show the unperturbed

resonances at $\omega_1$ and $\omega_2$, while the dashed curves show the shifted positions at $\omega_1'$ and $\omega_2'$. Now the essential point of the analogy is that the number of degrees of freedom of the system remains conserved during the introduction of the perturbation, and consequently there can remain no vestige of the unperturbed resonances; they have to disappear completely. While this analogy is suggestive that in the nuclear case one should not expect maxima in the cross section at both the collective and single-particle resonances, we believe that there is an essential difference in the two systems. This is that in the nuclear case there is a dense continuum of single-particle eigenstates, or effectively an infinite number of degrees of freedom in the language of the analogy. Consequently it is possible "to conserve degrees of freedom" and at the same time to introduce extra structure into the cross section.

The fact that the single-nucleon bound states are pushed up into the continuum and become virtual states is a result of the repulsion of the remaining hole in the $O^{16}$ core. This shows the importance of including the diagonal continuum-continuum matrix elements of the interaction. The question may properly be raised at this point as to the effect on the collective resonances of the change of the bound single-particle states into virtual levels in the continuum. In the standard shell-model treatments of the giant dipole resonance,[6,7] this aspect of the problem is ignored, and it might be supposed that the results of the calculations might be changed drastically when some of the zero order wave functions are changed from bound to virtual. We will show here, however, that this is not the case and that the collective levels are relatively insensitive to this change in the starting wave functions, as long as the virtual levels remain relatively sharp.

It will suffice to establish this result in the framework of the ordinary shell model. The generalization of the proof to the extended shell model is straightforward. We also restrict ourselves to one virtual single-particle level "v", in addition to the bound states "i", or "j". Now, the virtual level is nothing other than a manifestation of a certain behavior of the continuum states "c" in the vicinity of the virtual level energy $\varepsilon_v$. For a sufficiently narrow level we can ignore the difference between the actual energy of the continuum state and $\varepsilon_v$, since only states very near this energy are important for the collective problem. There exist, of course, continuum states for the entire positive energy scale, but these non-resonant wave functions will be negligibly small inside the nucleus, and therefore they will not be coupled to the collective vibrations. (This is, of course, not strictly true, or there would be no damping.) With this approximation to the continuum energies, we can write the shell model Schrödinger equation for the amplitudes in the collective wave function

$$\Psi = \sum_i A_i \Psi_i + \sum_c A_c \Psi_c \quad , \tag{3}$$

as

$$\varepsilon_i A_i + \sum_{j \neq i} H_{ij} A_j + \sum_c H_{ic} A_c = E A_i \tag{4}$$

$$\varepsilon_v A_c + \sum_j H_{cj} A_j = E A_c \tag{5}$$

Here $\Psi_i$ and $\Psi_c$ are the bound and continuum basis functions, respectively, (the latter normalized in some large spherical box),

and $H_{ij}$, etc., the matrix elements of the particle-hole interaction. $\epsilon_i$ are the bound single-particle energies (including as always the energy of the associated hole and also its diagonal interaction with the excited particle).

We now make use of the fact that the continuum wave functions at and near resonance all have the same funtional shape inside the nucleus, and that this function can be represented as the wave packet[9]

$$\Psi_v = \sum_c \phi_c \Psi_c. \tag{6}$$

Thus, inside the nucleus, the continuum wave $\Psi_c$ has a strength proportional to the complex conjugate of the overlap amplitude

$$\phi_c = (\Psi_c, \Psi_v). \tag{7}$$

Furthermore, normalization requires

$$\sum_c |\phi_c|^2 = 1. \tag{8}$$

Our goal is to replace the infinite set of equations (5) by a single equation based on the one function $\Psi_v$ rather than on all the different $\Psi_c$. This can be accomplished by noting that the second term of the left hand member of Eq. (5) serves to couple the continuum excitations to the rest of the wave function, and hence is a sort of "driving force" which sets the scale of the $A_c$. The strength of the coupling is dependent upon the matrix elements

$$H_{cj} = \phi_c H_{vj}, \tag{9}$$

showing that

$$A_c = \phi_c A_v, \tag{10}$$

where $A_v$ is some constant independent of c. But substitution into Eq. (3) and use of Eq. (6) give

$$\Psi = \sum_i A_i \Psi_i + A_v \sum_c \phi_c \Psi_c$$

$$= \sum_i A_i \Psi_i + A_v \Psi_v \quad . \qquad (11)$$

This establishes that $A_v$ can be considered as the probability amplitude associated with the expansion of the collective wave function into the virtual state v.

We can now proceed to make further simplifications based on Eqs. (8), (9), and (10):

$$\sum_c H_{ic} A_c = H_{iv} \sum_c \phi_c^* \phi_c A_v$$

$$= H_{iv} A_v \quad . \qquad (12)$$

Substitution of these equations into Eqs. (4) and (5) lead finally to

$$\varepsilon_i A_i + \sum_{j \neq i} H_{ij} A_j + H_{iv} A_v = E A_i \qquad (13)$$

$$\varepsilon_v A_v + \sum_j H_{vj} A_j = E A_v \qquad (14)$$

From the form of Eqs. (13) and (14) it follows that the virtual level can be treated as simply another bound state j and that the fact that it actually has negative binding energy can be ignored -- as was to be proved, thus justifying the calculations of references 6 and 7.

In summary, we hope to have conveyed here, albeit without the numerical details of the computations (for which the interested

reader can consult reference 1), the conviction that systematic application of the shell model gives not only a qualitative understanding but also a good quantitative account of the damping process in the giant dipole vibration in a nucleus.

TABLE 1

Damping of the 22-MeV Dipole State

| Emission Channel | *Neutron Partial Widths (MeV) | *Proton Partial Widths (MeV) |
|---|---|---|
| $s_{1/2}\ p^{-1}_{1/2}$ | 0.01 | 0.01 |
| $d_{3/2}\ p^{-1}_{1/2}$ | 0.44 | 0.53 |
| | $\Gamma_{n_o} = 0.45$ | $\Gamma_{p_o} = 0.54$ |
| $s_{1/2}\ p^{-1}_{3/2}$ | 0.03 | 0.08 |
| †$d_{5/2}\ p^{-1}_{3/2}$ | 0.16 | 0.58 |
| $d_{3/2}\ p^{-1}_{3/2}$ | 0.00 | 0.00 |
| | $\Gamma_{n_1} = 0.19$ | $\Gamma_{p_1} = 0.66$ |
| | $\Gamma_n = \Gamma_{n_o} + \Gamma_{n_1}$ | $\Gamma_p = \Gamma_{p_o} + \Gamma_{p_1}$ |
| | $= 0.64$ | $= 1.20$ |

Total width $= \Gamma = \Gamma_n + \Gamma_p$

$= 1.84$ MeV

($\Gamma_{exp} = 1.7$ Mev, Bolen and Whitehead, reference 4.)

* Subscripts 0 and 1 refer to ground state and excited state particle groups, respectively.

† In this channel the proximity of the single-particle resonance to the dipole state makes the calculation very sensitive to changes in the energy values used.

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[9] Here we lean heavily upon Fermi's treatment of the virtual level theory of alpha decay, [E. Fermi, "Nuclear Physics", p. 59, notes compiled by J. Orear, A. H. Rosenfeld, and R. A. Schluter, revised edition, University of Chicago Press (1950)].

## CAPTIONS FOR THE FIGURES

Fig. 1. $(\gamma, n)$ cross section in $O^{16}$ as a function of the gamma ray energy -- from Bolen and Whitehead (reference 4). Note the giant dipole resonance maxima at 22 and 24 MeV.

Fig. 2. Schematic single-nucleon energy level diagram for $O^{16}$. Note the positive energy continuum states for all different angular momenta (abscissa). As the core nucleons fill up the bound states through the p-level, only the bound s- and d-levels near the nucleon emission threshold are available for the dynamics of the collective giant dipole oscillation. (Spin-orbit splitting is neglected.) Excitation of a nucleon into an energy-conserving positive energy state leads to the escape of the nucleon and the damping of the vibration.

Fig. 3. Feynman diagram corresponding to the extended shell model description of a nuclear collective vibration. Time increases upwards, so that backward-pointing arrows correspond to holes in the core. A forward-going thin line indicates an excited nucleon in a bound single-nucleon state, while a heavy line indicates an unbound continuum state. The intermediate heavy line gives a real second order shift in the resonance, while a terminating heavy line corresponds to the matrix element for damping (imaginary second order shift in the resonance).

Fig. 4. Hamiltonian matrix divided into the usual shell-model part, acting only in the subspace of Hilbert space spanned by the bound single-nucleon states (S.M.), and the part acting in the subspace spanned by the continuum wave functions (lower right corner). H' connects the two subspaces and produces single-nucleon damping of the collective state. Note the approximation of neglecting the off-diagonal matrix elements between continuum states.

Fig. 5. Photonuclear cross section vs. gamma ray energy. The dashed line shows the cross section to be expected for direct excitation into the continuum (for constant matrix element and state density), while the curve gives a schematic representation of the interference in the photonuclear cross section between the resonant and direct processes (after Fano, reference 8). This interference distorts the Lorentzian Breit-Wigner type line which would be expected from the resonance alone and changes it into an asymmetric line with a long tail on one side. Under the circumstances prevailing in the nuclear problem, the relative phase is such that the tail falls on the high energy side, in agreement with experiment (see Fig. 1.).

Fig. 6. Cross section vs. frequency for a system of two harmonic oscillators. The solid curves show the resonances at the two unperturbed natural frequencies $\omega_1$ and $\omega_2$.

Introduction of a coupling between the oscillators causes the resonances to shift to the new positions (dashed curves) at $\omega_1'$ and $\omega_2'$. Thus the number of resonances is conserved under the application of an interaction. Although suggestive, this model is a misleading analogue to the nucleus, where the photonuclear cross section should exhibit resonances <u>both</u> of the single-nucleon type (unperturbed lines) and of the collective type (resulting from the nucleon-nucleon interaction).

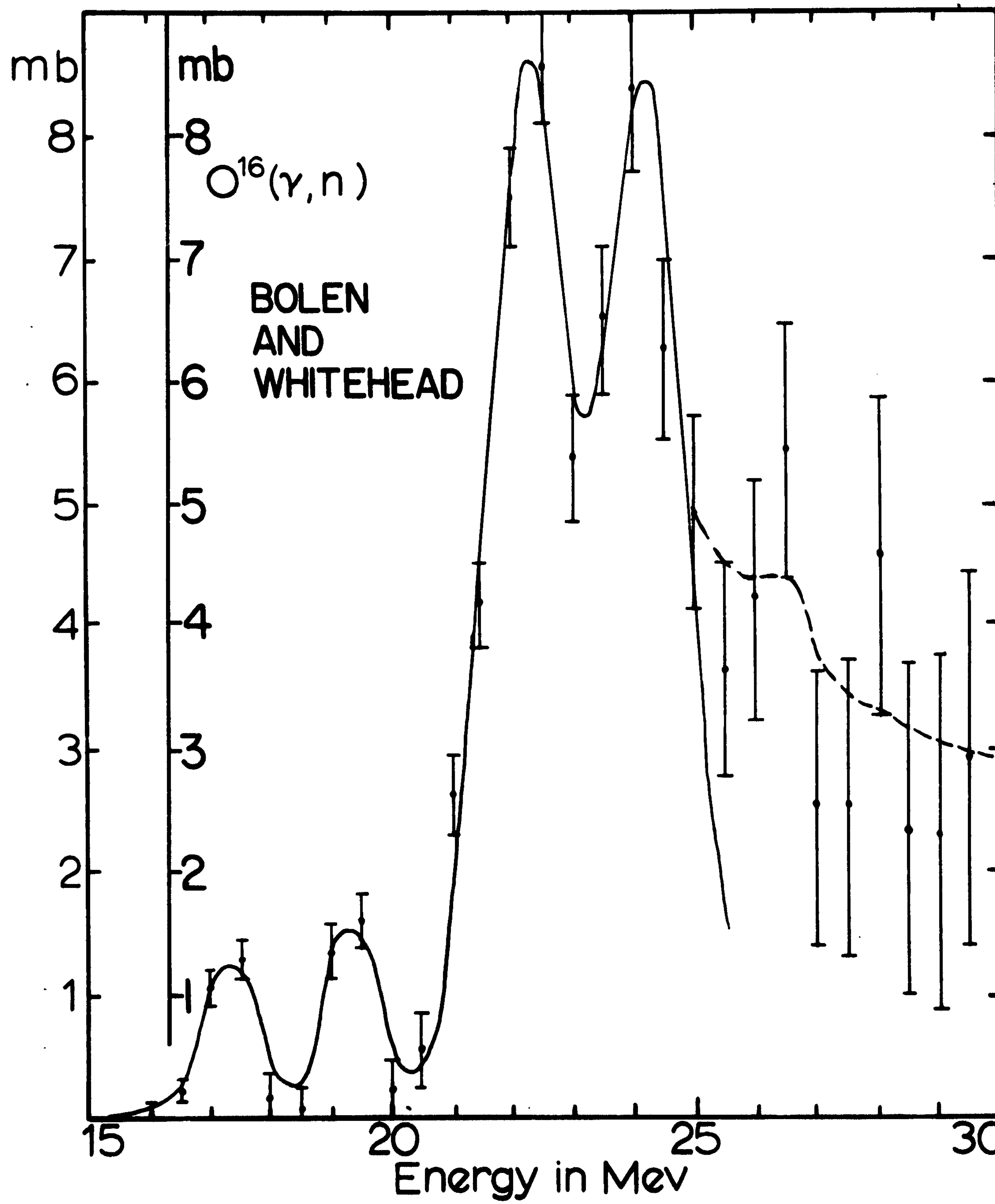
mb
mb
8
7
6
5
4
3
2
1
$O^{16}(\gamma, n)$
BOLEN
AND
WHITEHEAD
15
20
25
30
Energy in Mev

Fig. 1

FIGURE 2

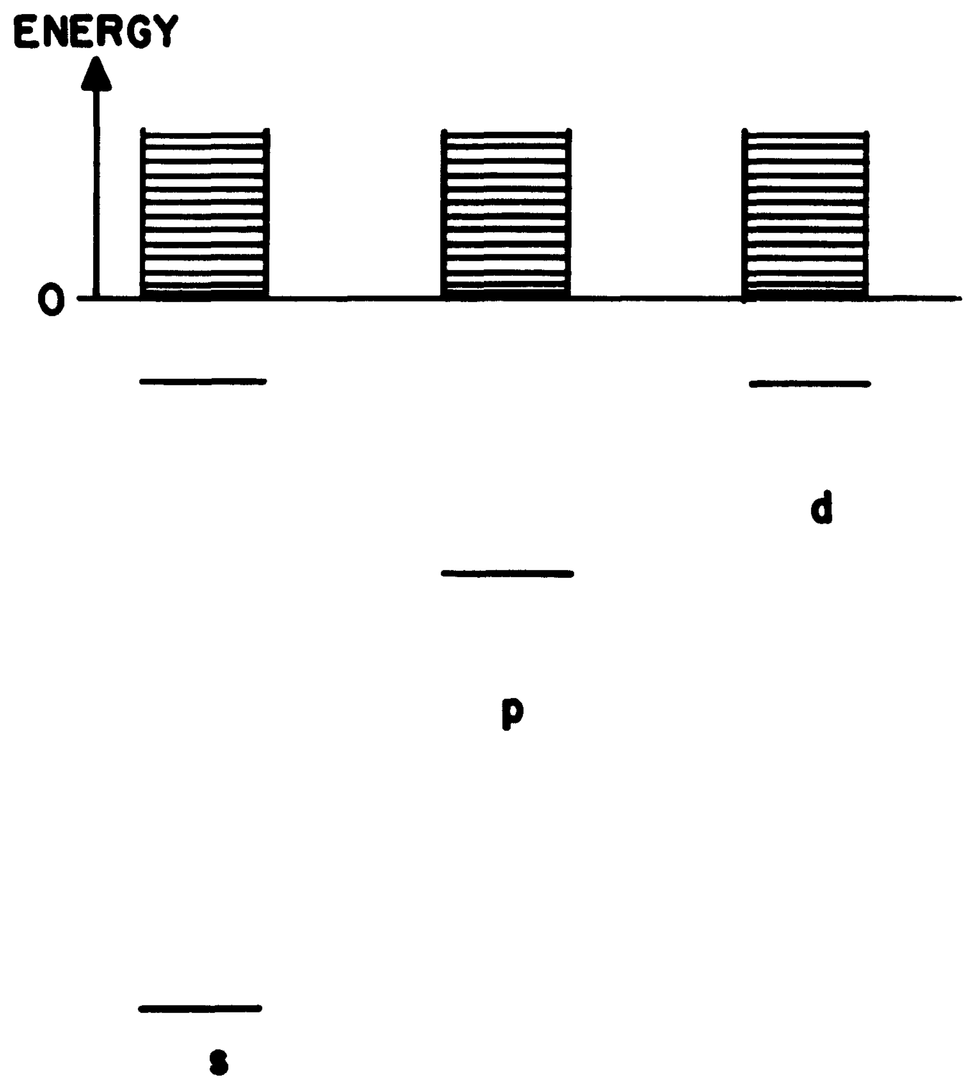

FIGURE 3

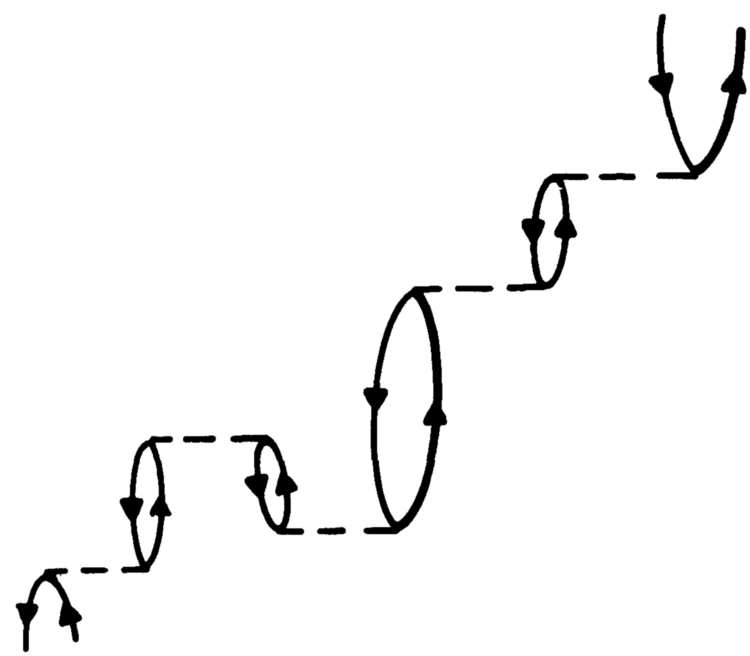

FIGURE 4

$$\left(\begin{array}{c|c} S.M & H' \\ \hline H' & \begin{matrix} X & O & \\ O & X & \\ & & \ddots \end{matrix} \end{array}\right)$$

FIGURE 5

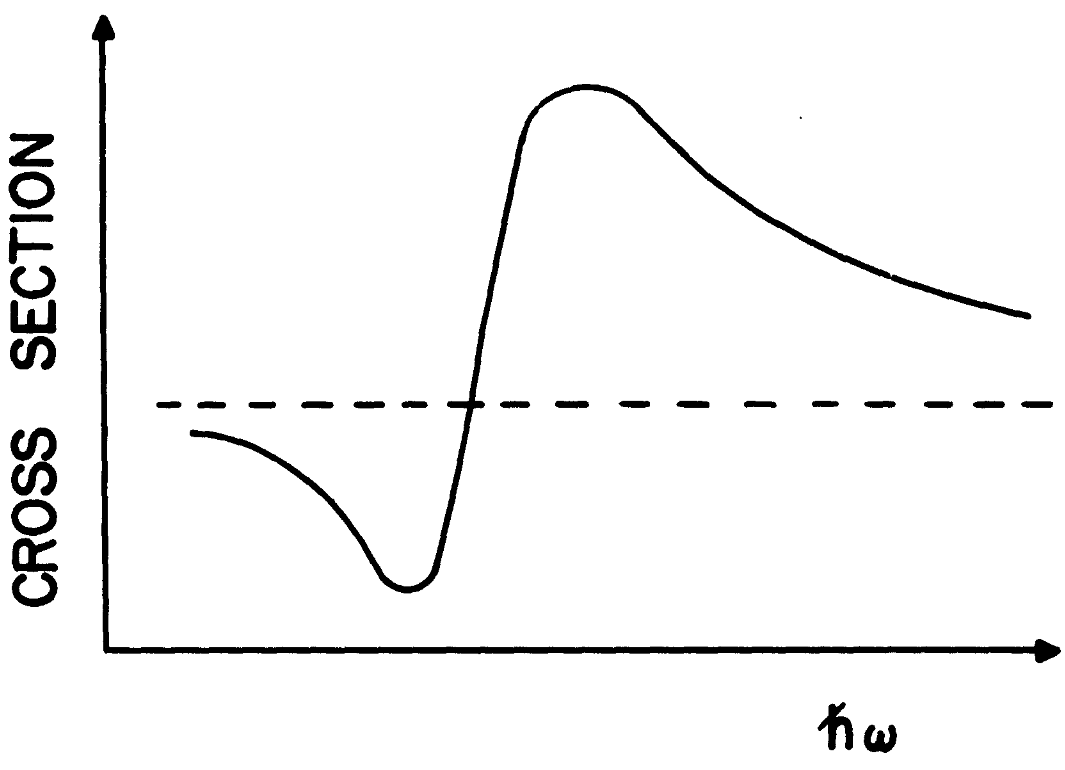

FIGURE 6